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On Lambert series and continued fractions

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Abstract: In this paper, we have established certain results involving Lambert series and continued fractions.

Keywords and Phrases: Lambert series, continued fractions, q-shifted factorial, basic hypergeometric series.
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1. Introduction, Notations and Definitions

The q-shifted factorial is defined by,

$$(a,q)_n = \begin{cases} 0 & \text{if } n = 0; \\ (1-a)(1-aq)(1-aq^2)\ldots,(1-aq^{n-1}) & \text{if } n \geq 1. \end{cases}$$

Also,

$$(a;q)_{-n} = \frac{q^{n(n+1)/2}}{(-a)^n(q/a;q)_n}.$$

The generalized basic hypergeometric series is given by

$${}_{r+1}\Phi_r \left[\begin{array}{c} a_1, a_2, ..., a_{r+1}; q; z \\ b_1, b_2, ..., b_r \end{array} \right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, ..., a_{r+1}; q)_n z^n}{(q, b_1, b_2, ..., b_r; q)_n},$$

where $(a_1, a_2, ..., a_{r+1}; q)_n = (a_1; q)_n (a_2; q)_n ... (a_{r+1}; q)_n$ and max.$(|z|, |q|) < 1$ for the convergence of the series.

The generalised bilateral basic hypergeometric series is defined as,

$${}_r\Psi_r \left[\begin{array}{c} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_r \end{array} \right] = \sum_{n=-\infty}^{\infty} \frac{(a_1, a_2, ..., a_r; q)_n z^n}{(b_1, b_2, ..., b_r; q)_n},$$

where $\left| \frac{b_1, b_2, ..., b_r}{a_1, a_2, ..., a_r} \right| < |z| < 1$ for the convergence of the series.
An expression of the form,

$$b_0 + \frac{a_1}{b_1+} \frac{a_2}{b_2+} \frac{a_3}{b_3 + ...}$$